

# MATHEMATICS LECTURES FOR IIT-JEE BY MANISH KALIA

## Matrices

### MCQ-Single Correct

1. If  $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$ , then  $\text{adj}(3A^2 + 12A)$  is equal to:

(1)  $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

(2)  $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

(3)  $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

(4)  $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

[2017]

2. If  $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$  and  $A \text{adj} A = AA^T$ , then  $5a + b$  is equal to :

(1) 5

(2) 4

(3) 13

(4) -1

[2016]

3. If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$  is a matrix satisfying the equation  $AA^T = 9I$ , where  $I$  is  $3 \times 3$  identity

matrix, then ordered pair  $(a, b)$  is equal to :

(1)  $(-2, 1)$

(2)  $(2, 1)$

(3)  $(-2, -1)$

(4)  $(2, -1)$

[2015]

4. If  $A$  is a  $3 \times 3$  non-singular matrix such that  $AA' = A'A$  and  $B = A^{-1}A$ , then  $BB'$  equals

(1)  $I + B$

(2)  $I$

(3)  $B^{-1}$

(4)  $(B^{-1})'$

[2014]

5. If  $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$  is the adjoint of a  $3 \times 3$  matrix  $A$  and  $|A| = 4$ , then  $\alpha$  is equal to

(1) 11

(2) 5

(3) 0

(4) 4

[2013]

6. Let  $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$ . If  $u_1$  and  $u_2$  are column matrices such that  $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$  and  $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ , then

$u_1 + u_2$  is equal to

# MATHEMATICS LECTURES FOR IIT-JEE BY MANISH KALIA

(1)  $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

(2)  $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

(3)  $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

(4)  $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$

[2012]

7. If  $\omega \neq 1$  is the complex cube root of unity and matrix  $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$ , then  $H^{70}$  is equal to

(1)  $H^2$

(2)  $H$

(3)  $0$

(4)  $-H$

[2011]

8. The number of  $3 \times 3$  non-singular matrices, with four entries as 1 and all other entries as 0, is

(1) 5

(2) 6

(3) at least 7

(4) less than 4

[2010]

9. Let  $A$  be a square matrix all of whose entries are integers. Then which one of the following is true?

(1) If  $\det A = \pm 1$ , then  $A^{-1}$  exists but all its entries are not necessarily integers

(2) If  $\det A = \pm 1$ , then  $A^{-1}$  exists and all its entries are non-integers

(3) If  $\det A = \pm 1$ , then  $A^{-1}$  exists and all its entries are integers

(4) If  $\det A = \pm 1$ , then  $A^{-1}$  need not exist

[2008]

10. Let  $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ . If  $|A^2| = 25$ , then  $|\alpha|$  equals

(1)  $5^2$

(2) 1

(3)  $1/5$

(4) 5

[2007]

11. If  $A$  and  $B$  are square matrices of size  $n \times n$  such that  $A^2 - B^2 = (A - B)(A + B)$ , then which of the following will be always true?

(1)  $A = B$

(2)  $AB = BA$

(3) Either of  $A$  or  $B$  is a zero matrix

(4) Either of  $A$  or  $B$  is an identity matrix

[2006]

12. Let  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$  and  $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ ,  $a, b \in \mathbb{N}$ . Then

(1) there cannot exist any  $B$  such that  $AB = BA$

# MATHEMATICS LECTURES FOR IIT-JEE BY MANISH KALIA

(2) there exist more than one but finite number of B's such that  $AB = BA$

(3) there exists exactly one B such that  $AB = BA$

(4) there exist infinitely many B's such that  $AB = BA$

[2006]

13. If  $A^2 - A + I = 0$ , then the inverse of A is

(1)  $A + I$

(2)  $A$

(3)  $A - I$

(4)  $I - A$

14. If  $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$  and  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , then which one of the following holds for all  $n \geq 1$ , by the principle of mathematical induction

(1)  $A^n = nA - (n-1)I$

(2)  $A^n = 2^{n-1}A - (n-1)I$

(3)  $A^n = nA + (n-1)I$

(4)  $A^n = 2^{n-1}A + (n-1)I$

[2005]

15. Let  $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$ . The only correct statement about the matrix A is

(1) A is a zero matrix

(2)  $A^2 = I$

(3)  $A^{-1}$  does not exist

(4)  $A = (-1)I$ , where I is a unit matrix

[2004]

16. Let  $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$  (10) and  $B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$ . If B is the inverse of matrix A, then  $\alpha$  is

(1) -2

(2) 5

(3) 2

(4) -1

[2004]

17. If  $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$  and  $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$ , then

(1)  $\alpha = a^2 + b^2, \beta = ab$

(2)  $\alpha = a^2 + b^2, \beta = 2ab$

(3)  $\alpha = a^2 + b^2, \beta = a^2 - b^2$

(4)  $\alpha = 2ab, \beta = a^2 + b^2$

[2003]

## Assertion-Reason type

1. Consider the following relation **R** on the set of real square matrices of order 3. [2011]

$R = \{(A, B) | A = P^{-1}BP \text{ for some invertible matrix } P\}$ .

**Statement - I** : R is an equivalence relation.

**Statement-II** : For any two invertible  $3 \times 3$  matrices M and N,  $(MN)^{-1} = N^{-1}M^{-1}$ .

## MATHEMATICS LECTURES FOR IIT-JEE BY MANISH KALIA

- (1) Statement-I is True; Statement-II is true; Statement-II is **not** a correct explanation of Statement-I.
- (2) Statement-I is True; Statement-II is False.
- (3) Statement-I is False; Statement-II is true
- (4) Statement-I is True; Statement-II is true; Statement-II is a **correct** explanation of Statement-I.
2. Let  $A$  be a  $2 \times 2$  matrix with non-zero entries and let  $A^2 = I$ , where  $I$  is  $2 \times 2$  identity matrix. Define  $\text{Tr}(A)$  = sum of diagonal elements of  $A$  and  $|A|$  = determinant of matrix  $A$ . **[2010]**  
**Statement-I** :  $\text{Tr}(A) = 0$   
**Statement-II** :  $|A| = 1$
- (1) Statement-I is True; Statement-II is true; Statement-II is **not** a correct explanation of Statement-I.
- (2) Statement-I is True; Statement-II is False.
- (3) Statement-I is False; Statement-II is true
- (4) Statement-I is True; Statement-II is true; Statement-II is a **correct** explanation of Statement-I.
3. Let  $A$  be a  $2 \times 2$  matrix **[2009]**  
**Statement-I** :  $\text{adj}(\text{adj } A) = A$   
**Statement-II** :  $|\text{adj } A| = |A|$
- (1) Statement-I is True; Statement-II is true; Statement-II is **not** a correct explanation of Statement-I.
- (2) Statement-I is True; Statement-II is False.
- (3) Statement-I is False; Statement-II is true
- (4) Statement-I is True; Statement-II is true; Statement-II is a **correct** explanation of Statement-I.
4. Let  $A$  be a  $2 \times 2$  matrix with real entries. Let  $I$  be the  $2 \times 2$  identity matrix. Denote by  $\text{tr}(A)$ , the sum of diagonal entries of  $A$ . Assume that  $A^2 = I$  **[2008]**  
**Statement-I** : If  $A \neq I$  and  $A \neq -I$ , then  $\det A = -1$ .  
**Statement-II** : If  $A \neq I$  and  $A \neq -I$ , then  $\text{tr}(A) \neq 0$
- (1) Statement-I is True; Statement-II is true; Statement-II is **not** a correct explanation of Statement-I.
- (2) Statement-I is True; Statement-II is False.
- (3) Statement-I is False; Statement-II is true
- (4) Statement-I is True; Statement-II is true; Statement-II is a **correct** explanation of Statement-I.