Limits Continuity and Differentiability

JEE-MAINS (PREVIOUS YEAR)

MCQ-Single Correct

1. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}$. g(x), then g(x) equals :

(1) $\frac{9}{1+9x^3}$ (2) $\frac{3x\sqrt{x}}{1-9x^3}$

(3) $\frac{3x}{1-9x^3}$ (4) $\frac{3}{1+9x^3}$ **[2017]**

2. $\lim\limits_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals :

(1) $\frac{1}{24}$ (2) $\frac{1}{16}$

(3) 1 (4) $\frac{1}{4}$ **[2017]**

3. Let $p = \lim\limits_{x \to 0+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to :

(1) 1 (2) ½

(3) ¼ (4) 2 **[2016]**

4. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and g(x) = f(f(x)), then :

(1) g'(0) = cos(log2)

(2) g'(0) = -cos(log2)

(3) g is differentiable at x = 0 and g'(0) = - sin(log2)

(4) g is not differentiable at x = 0 **[2016]**

5. $\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to :

(1) 3 (2) 2

(3) ½ (4) 4 **[2015]**

6. If the function, g(x) = $\begin{cases} k\sqrt{x+1} & ,0\le x\le 3 \\ mx+2 & ,3<x\le 5 \end{cases}$ is differentiable, then the value of k + m is :

(1) 16/5 (2) 10/3

(3) 4 (4) 2 **[2015]**

7. $\lim\limits_{x\to 0}\dfrac{\sin\left(\pi\cos^2 x\right)}{x^2}$ is equal to

(1) $\pi/2$ (2) 1

(3) - π (4) π **[2014]**

8. If $y=\sec\left(\tan^{-1}x\right)$, then $\dfrac{dy}{dx}$ at x = 1 is equal to

(1) ½ (2) 1

(3) $\sqrt{2}$ (4) $\dfrac{1}{\sqrt{2}}$ **[2013]**

9. $\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to :

(1) 1/2 (2) 1

(3) 2 (4) -1/4 **[2013]**

10. Let f: $R\to[0,\infty)$ be such that $\lim\limits_{x\to 5}f(x)$ exists and $\lim\limits_{x\to 5}\dfrac{(f(x))^2-9}{\sqrt{|x-5|}}=0$, then $\lim\limits_{x\to 5}f(x)$ equals

(1) 2 (2) 3

(3) 0 (4) 1 **[2011]**

11. If function f(x) is differentiable at x = a, then $\lim_{x\to a}\frac{x^2 f(a)-a^2 f(x)}{x-a}$ is

(1) $2af(a)-a^2 f'(a)$ (2) $2af(a)+a^2 f(a)$

(3) $-a^2 f'(a)$ (4) $af(a)-a^2 f'(a)$ **[2011]**

12. Let f : $R\to R$ be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)}=1$. Then $\lim_{x\to\infty}\frac{f(2x)}{f(x)}=$

(1) 2/3 (2) 3/2

(3) 3 (4) 1 **[2010]**

13. Let f : $(-1,1)\to R$ be a differentiable function with f(0) = -1 and f'(0) =1. Let $g(x)=[f(2f(x)+2)]^2$. Then g'(0) =

(1) -4 (2) 0

(3) -2 (4) 4 **[2010]**

14. Let y be an implicit function of x defined by $x^{2x}-2x^x\cot y-1=0$. Then y'(1) equals

(1) -1 (2) 1

(3) log 2 (4) - log 2 **[2009]**

15. Let f(x) = $\begin{cases}(x-1)\sin\left(\frac{1}{x-1}\right), & if\ x\neq 1\\ 0, & if\ x=1\end{cases}$. Then which one of the following is true?

(1) f is neither differentiable at x = 0 nor at x = 1

(2) f is differentiable at x = 0 and at x = 1

(3) f is differentiable at x = 0 but not at x = 1

(4) f is differentiable at x = 1 but not at x = 0 **[2008]**

16. The set of points where $f(x)=\frac{x}{1+|x|}$ is differentiable is

(1) $(-\infty, 0) \cup (0, \infty)$

(2) $(-\infty, -1) \cup (-1, \infty)$

(3) $(-\infty, \infty)$

(4) $(0, \infty)$ **[2006]**

17. If $x^m . y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is

(1) $\frac{y}{x}$

(2) $\frac{x+y}{xy}$

(3) xy

(4) $\frac{x}{y}$ **[2006]**

18. Let α and β be the distant roots of $ax^2 + bx + c = 0$, then $\lim\limits_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to

(1) $\frac{a^2}{2}(\alpha - \beta)^2$

(2) 0

(3) $-\frac{a^2}{2}(\alpha - \beta)^2$

(4) $\frac{1}{2}(\alpha - \beta)^2$ **[2005]**

19. Suppose f(x) is differentiable x = 1 and $\lim\limits_{h \to 0} \frac{1}{h} f(1+h) = 5$, then f'(1) equals

(1) 3

(2) 4

(3) 5

(4) 6 **[2005]**

20. Let f be differentiable for all x. If f(1) = -2 and $f'(x) \geq 2$ for $x \in [1, 6]$, then

(1) $f(6) \geq 8$

(2) $f(6) < 8$

(3) f(6) < 5

(4) f(6) = 5 **[2005]**

21. If f is a real-valued differentiable function satisfying $|f(x)-f(y)| \le (x-y)^2$, x , y ε R and f(0) = 0 , then f(1) equals

(1) -1 (2) 0

(3) 2 (4) 1 **[2005]**

22. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b, are

(1) $a \in R, b \in R$ (2) a = 1, $b \in R$

(3) $a \in R, b = 2$ (4) a = 1 and b = 2 **[2004]**

23. Let $f(x) = \frac{1-\tan x}{4x-\pi}$, $x \neq \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right]$. If f(x) is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

(1) 1 (2) ½

(3) -1/2 (4) -1 **[2004]**

24. If $x = e^{y+e^{y+....to\,\infty}}$, x > 0 , then $\frac{dy}{dx}$ is

(1) $\frac{x}{1+x}$ (2) $\frac{1}{x}$

(3) $\frac{1-x}{x}$ (4) $\frac{1+x}{x}$ **[2004]**

25. $\lim_{x\to\pi/2}\frac{\left[1-\tan\left(\frac{x}{2}\right)\right]\left[1-\sin x\right]}{\left[1+\tan\left(\frac{x}{2}\right)\right]\left[\pi-2\pi\right]^3}$ is

(1) 1/8 (2) 0

(3) 1/32 (4) ∞ **[2003]**

26. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x} = k$, the value of k is

(1) 0 (2) -1/3

(3) 2/3 (4) -2/3 **[2003]**

27. Let f (a) = g(a) = k and their n^{th} derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n.

Further if $\lim_{x \to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4,$ then the value of k is

(1) 4 (2) 2

(3) 1 (4) 0 **[2003]**

28. If f(x) = $\begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & x \neq 0 \\ 0 & x = 0 \end{cases}$

then f(x) is

(1) Continuous as well as differentiable for all x
(2) Continuous for all x but not differentiable at x = 0
(3) Neither differentiable nor continuous at x = 0
(4) Discontinuous everywhere **[2003]**

29. $\lim_{n \to \infty} \frac{1+2^4+3^4+.....+n^4}{n^5} - \lim_{n \to \infty} \frac{1+2^3+3^3+......+n^3}{n^5}$

is

(1) $\frac{1}{30}$ (2) zero

(3) $\frac{1}{4}$ (4) $\frac{1}{5}$ **[2003]**

30. $\lim_{x \to 0} \frac{\log x^n - [x]}{[x]}$, $n \in N$, ([x] denotes greatest integer less than or equal to x)

(1) has value -1 (2) has value 0

(3) has value 1 (4) does not exist **[2002]**

31. $\lim_{n \to \infty} \frac{1^p+2^p+3^p+.....+n^p}{n^{p+1}}$ is

(1) $\frac{1}{p+1}$ (2) $\frac{1}{p-1}$

(3) $\frac{1}{p}-\frac{1}{p-1}$ (4) $\frac{1}{p+2}$ **[2002]**

32. f is defined in [-5,5] as f(x) = x, if x is rational and = -x, if x is irrational. Then

(1) f(x) is continuous at every x, except x = 0

(2) f(x) is discontinuous at every x, except x = 0

(3) f(x) is continuous everywhere

(4) f(x) is discontinuous everywhere

33. If $y=\left(x+\sqrt{1+x^2}\right)^x$, then $\left(1+x^2\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}$ is

(1) n^2y (2) $-n^2y$

(3) - y (4) $2n^2y$ **[2002]**

34. If f(1) = 1, f'(1) = 2, then $\lim_{x\to 1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ is

(1) 2 (2) 4

(3) 1 (4) ½ **[2002]**

35. $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is

(1) 1 (2) -1

(3) 0 (4) does not exist **[2002]**

36. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^{1/x}$

(1) e^4 (2) e^2

(3) e^3 (4) 1 **[2002]**

37. Let f(x) = 4 and f′(x) = 4 , then $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$ equals

(1) 2 (2) -2

(3) -4 (4) 3 **[2002]**

Assertion – Reason Type

1. Define F (x) as the product of two real functions $f_1(x) = x,\ \ x \in R$ and **[2011]**

$$f_2(x) = \begin{cases} \sin 1/x, & if\ x \neq 0 \\ 0, & if\ x = 0 \end{cases}$$ as follows

F (x) = $\begin{cases} f_1(x).f_2(x), & if\ x \neq 0 \\ 0, & if\ x = 0 \end{cases}$

Statement-I : F(x) is continuous on R

Statement-II : $f_1(x)$ and $f_2(x)$ are continuous on R

2. Let $f : R \to R$ be a continuous function defined by $f(x) = \dfrac{1}{e^x + 2e^{-x}}$, **[2010]**

Statement-I : $f(c) = \dfrac{1}{3}$, for some $c \in R$

Statement-II : $0 < f(x) \leq \dfrac{1}{2\sqrt{2}}$, for all $x \in R$

3. Let f(x) = x|x| and g(x) = sin x **[2009]**

Statement-I : gof is differentiable at x = 0 and its derivative is continuous at that point.

Statement-II : gof is twice differentiable at x = 0.