Definite Integrals

JEE-MAINS (PREVIOUS YEAR)

MCQ-Single Correct

1. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to

(1) -2 (2) 2

(3) 4 (4) -1 **[2017]**

2. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)......3n}{n^{2n}}\right)^{1/n}$ is equal to :

(1) $\frac{27}{e^2}$ (2) $\frac{9}{e^2}$

(3) 3log3 -2 (4) $\frac{18}{e^4}$ **[2016]**

3. The integral $\int_2^4 \frac{\log x^2}{\log x^2+\log\left(36-12x+x^2\right)}dx$ is equal to :

(1) 4 (2) 1

(3) 6 (4) 2 **[2015]**

4. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}dx$ equals

(1) $\pi-4$ (2) $\frac{2\pi}{3}-4-4\sqrt{3}$

(3) $4\sqrt{3}-4$ (4) $4\sqrt{3}-4-\frac{\pi}{3}$ **[2014]**

5. If g(x) = $\int_0^x \cos 4t\,dt$ then g(x + π) equals

(1) g(x) (2) g(x) . g(π)

(3) $\dfrac{g(x)}{g(\pi)}$ (4) g(x) + g(π) **[2012]**

6. Let [.] denotes the greatest integer function, then the value of $\int_0^{1.5} x\left[x^2\right]dx$ is

(1) ¾ (2) 5/4

(3) 0 (4) 3/2 **[2011]**

7. Let p(x) be a function defined on R such that p'(x) = p'(1-x), for all x ε [0,1] , p(0) = 1 and p(1) = 41

. Then $\int_0^1 p(x)dx$ equals

(1) 21 (2) 41

(3) 42 (4) $\sqrt{41}$ **[2010]**

8. $\int_0^{\pi}\left[\cot x\right]dx$, [*] denotes the greatest integer function, is equal to

(1) $\dfrac{\pi}{2}$ (2) 1

(3) -1 (4) $-\dfrac{\pi}{2}$ **[2009]**

9. Let I = $\int_0^1 \dfrac{\sin x}{\sqrt{x}}dx$ and J = $\int_0^1 \dfrac{\cos x}{\sqrt{x}}dx$. Then which one of the following is true?

(1) I > 2/3 and J > 2 (2) I < 2/3 and J < 2

(3) I < 2/3 and J > 2 (4) I > 2/3 and J < 2 **[2008]**

10. The value of the integral, $\int_3^6 \dfrac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is

(1) ½ (2) 3/2

(3) 2 (4) 1 **[2006]**

11. $\int_0^{\pi} xf(\sin x)dx$ is equal to **[2006]**

(1) $\pi\int_0^{\pi} f(\cos x)dx$ (2) $\pi\int_0^{\pi} f(\sin x)dx$

(3) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$ (4) $\pi\int_0^{\pi/2} f(\cos x)dx$

12. $\int_{-3\pi/2}^{-\pi/2}\left[(x+\pi)^3+\cos^2(x+3\pi)\right]dx$ is equal to

(1) $\frac{\pi^4}{32}$ (2) $\frac{\pi^4}{32}+\frac{\pi}{2}$

(3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}-1$ **[2006]**

13. The value of $\int_1^a [x] f'(x)dx$, a > 1, where [x] denotes the greatest integer not exceeding x is

(1) af(a) – {f(1) +f(2)+……+f([a])} (2) [a] f(a) – {f(1) +f(2) + ….. +f([a])}

(3) [a]f([a]) – {f(1) + f(2) + ….+f(a)} (4) af([a]) – {f(1) + f(2) + …..+f(a)}**[2006]**

14. $\lim_{n\to\infty}[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+.....+\frac{1}{n^2}\sec^2 1]$ equals

(1) $\frac{1}{2}\sec 1$ (2) $\frac{1}{2}\operatorname{cosec} 1$

(3) tan1 (4) $\frac{1}{2}\tan 1$ **[2005]**

15. If $I_1=\int_0^1 2^{x^2}dx$, $I_2=\int_0^1 2^{x^3}dx$, $I_3=\int_1^2 2^{x^2}dx$ and $I_4=\int_1^2 2^{x^3}dx$ then

(1) $I_2 > I_1$ (2) $I_1 > I_2$

(3) $I_3 = I_4$ (4) $I_3 > I_4$ **[2005]**

16. Let f : $R\to R$ be a differentiable function having f(2) = 6 , f'(2) = $\left(\frac{1}{48}\right)$. Then $\lim_{x\to 2}\int_6^{f(x)}\frac{4t^3}{x-2}dt$ equals

(1) 24 (2) 36

(3) 12 (4) 18 **[2005]**

17. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, a > 0 , is

(1) aπ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{a}$ (4) 2π **[2005]**

18. $\lim_{n\to\infty} \sum_{r-1}^{n} \frac{1}{n} e^{\frac{r}{n}}$ is

(1) e (2) e - 1

(3) 1-e (4) e + 1 **[2004]**

19. The value of $\int_{-2}^{3} |1-x^2| dx$ is

(1) $\frac{28}{3}$ (2) $\frac{14}{3}$

(3) $\frac{7}{3}$ (4) $\frac{1}{3}$ **[2004]**

20. The value of $I = \int_{0}^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1+\sin 2x}} dx$ is

(1) 0 (2) 1

(3) 2 (4) 3 **[2004]**

21. If $\int_{0}^{\pi} xf(\sin x)dx = A\int_{0}^{\pi/2} f(\sin x)dx$, then A is

(1) 0 (2) π

(3) π/4 (4) 2π **[2004]**

22. If $f(x)=\dfrac{e^x}{1+e^x}$, $I_1=\displaystyle\int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and $I_2=\displaystyle\int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$ then the value of $\dfrac{I_2}{I_1}$ is

(1) 2
(2) -3
(3) -1
(4) 1 **[2004]**

23. If f(y) = e^y , g(y) = y ; y > 0 and F(t) = $\displaystyle\int_0^t f(t-y)g(y)dy$, then

(1) F(t) = $1-e^{-t}(1+t)$
(2) F(t) = $e^t-(1+t)$
(3) F(t) = te^t
(4) F(t) = te^{-t} **[2003]**

24. If f(a + b –x) = f(x), then $\displaystyle\int_a^b xf(x)dx$ is equal to

(1) $\dfrac{a+b}{2}\displaystyle\int_a^b f(b-x)dx$
(2) $\dfrac{a+b}{2}\displaystyle\int_a^b f(x)dx$
(3) $\dfrac{b-a}{2}\displaystyle\int_a^b f(x)dx$
(4) $\dfrac{a+b}{2}\displaystyle\int_a^b f(a+b-x)dx$ **[2003]**

25. The value of $\displaystyle\lim_{x\to 0}\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}$ is

(1) 3
(2) 2
(3) 1
(4) 0 **[2003]**

26. The value of the integral $I=\displaystyle\int_0^1 x(1-x)^n\,dx$ is

(1) $\dfrac{1}{n+1}$
(2) $\dfrac{1}{n+2}$
(3) $\dfrac{1}{n+1}-\dfrac{1}{n+2}$
(4) $\dfrac{1}{n+1}+\dfrac{1}{n+2}$ **[2003]**

27. Let $\frac{d}{dx}F(x)=\left(\frac{e^{\sin x}}{x}\right), x>0$. If $\int_1^4 \frac{3}{x}e^{\sin x^3}dx = F(k)-F(1)$, then one of the possible values of k, is

(1) 15 (2) 16

(3) 63 (4) 64 **[2003]**

28. Let f(x) be a function satisfying f'(x) = f(x) with f(0) = 1 and g(x) be a function that satisfies f(x) + g(x) = x^2. Then the value of the integral $\int_0^1 f(x)g(x)dx$, is

(1) $e-\frac{e^2}{2}-\frac{5}{2}$ (2) $e+\frac{e^2}{2}-\frac{3}{2}$

(3) $e-\frac{e^2}{2}-\frac{3}{2}$ (4) $e+\frac{e^2}{2}+\frac{5}{2}$ **[2003]**

29. $\int_0^{\sqrt{2}}\left[x^2\right]dx$ is

(1) 2 - $\sqrt{2}$ (2) $2+\sqrt{2}$

(3) $\sqrt{2}-1$ (4) $\sqrt{2}-2$ **[2002]**

30. $I_n = \int_0^{\pi/4}\tan^n x dx$, then $\lim_{n\to\infty}\left[I_n+I_{n-2}\right]$ equals (1) ½

(2) 1

(3) ∞ (4) 0 **[2002]**

31. $\int_\pi^{10\pi}|\sin x|\,dx$ is

(1) 20 (2) 8

(3) 10 (4) 18 **[2002]**

32. If y = f(x) makes positive intercept of 2 and 0 unit in x and y axes and encloses an area of ¾ square units with the axes then $\int_0^2 f'(x)dx$ is

(1) 3/2 (2) 1

(3) 5/4 (4) -3/4 **[2002]**

33. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is

(1) $\frac{\pi^2}{4}$ (2) π^2

(3) 0 (4) $\frac{\pi}{2}$ **[2002]**

Assertion – Reason Type

1. **Statement – I** : The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$. **[2013]**

Statement – II : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.