TOPIC -Applications of Derivatives

JEE-MAINS (PREVIOUS YEAR)

MCQ-Single Correct

1. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower bed is

(1) 12.5 (2) 10
(3) 25 (4) 30 **[2017]**

2. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the y-axis passes through the point :

(1) $\left(-\frac{1}{2},-\frac{1}{2}\right)$ (2) $\left(\frac{1}{2},\frac{1}{2}\right)$
(3) $\left(\frac{1}{2},-\frac{1}{3}\right)$ (4) $\left(\frac{1}{2},\frac{1}{3}\right)$ **[2017]**

3. A wire of length 2 units is cut into two parts which are bent respectively to form to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then :

(1) $(4-\pi)x=\pi r$ (2) x = 2r
(3) 2x = r (4) 2x = $(\pi+4)r$ **[2016]**

4. Consider f(x) = $\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$, $x\in\left(0,\frac{\pi}{2}\right)$. A normal to y = f(x) at $x=\frac{\pi}{6}$ also passes through the point :

(1) $\left(0,\frac{2\pi}{3}\right)$ (2) $\left(\frac{\pi}{6},0\right)$
(3) $\left(\frac{\pi}{4},0\right)$ (4) (0,0) **[2016]**

5. Let f(x) be a polynomial of degree four having extreme values at x = 1 and x = 2 .If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$, then f(2) is equal to :

(1) -4 (2) 0
(3) 4 (4) -8 **[2015]**

6. The normal to the curve, $x^2+2xy-3y^2=0$, at (1,1) :

(1) meets the curve again in the second quadrant.
(2) meets the curve again in the third quadrant.
(3) meets the curve again in the fourth quadrant.
(4) does not meet the curve again. **[2015]**

7. If x = -1 and x = 2 are extreme points of f(x) = $\alpha \log|x| + \beta x^2 + x$, then

(1) $\alpha = -6, \beta = \frac{1}{2}$
(2) $\alpha = -6, \beta = -\frac{1}{2}$
(3) $\alpha = 2, \beta = -\frac{1}{2}$
(4) $\alpha = 2, \beta = \frac{1}{2}$ **[2014]**

8. If f and g are differentiable functions in [0,1] satisfying f(0) = 2 = g(1), g(0) = 0 and f(1) = 6, then for some $c \in]0,1[$

(1) 2f′(c) = g′(c)
(2) 2 f′(c) = 3 g′(c)
(3) f′(c) = g′(c)
(4) f′(c) = 2 g′(c) **[2014]**

9. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0,1]

(1) lies between 2 and 3
(2) lies between -1 and 0
(3) does not exist
(4) lies between 1 and 2 **[2013]**

10. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t|dt$, $x \in R$, which are parallel to the line y = 2x , are equal to

(1) ± 2
(2) ± 3
(3) ± 4
(4) ± 1 **[2013]**

11. The curve that passes through the point (2,3) , and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact, is given by

(1) $x^2 + y^2 = 13$
(2) $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 2$
(3) 2y -3x = 0
(4) $y = \frac{6}{x}$ **[2011]**

12. Let f : $R \to R$ be defined by f(x) = $\begin{cases} k - 2x, & if\ x \le -1 \\ 2x + 3, & if\ x > -1 \end{cases}$. If f has a local minimum at x = -1 , then a possible value of k is

(1) 0
(2) -1/2
(3) -1
(4) 1 **[2010]**

13. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is

(1) y = 1
(2) y = 2
(3) y = 3
(4) y = 0 **[2010]**

14. Given P(x) = $x^4 + ax^3 + bx^2 + cx + d$ such that x = 0 is the only real root of P′(x) = 0 . If P(-1) < P(1) , then in the interval [-1,1]

(1) P(-1) is the minimum and P(1) is the maximum of P
(2) P(-1) is not minimum but P(1) is the maximum of P
(3) P(-1) is the minimum and P(1) is not the maximum of P
(4) neither P(-1) is the minimum nor P(1) is the maximum of P **[2009]**

15. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is

(1) $\frac{3\sqrt{2}}{8}$ (2) $\frac{2\sqrt{3}}{8}$

(3) $\frac{3\sqrt{2}}{5}$ (4) $\frac{\sqrt{3}}{4}$ **[2009]**

16. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?

(1) 7 (2) 1

(3) 3 (4) 5 **[2008]**

17. Suppose the cube $x^3 - px + q$ has three distinct real roots where p > 0 and q > 0. Then which one of the following holds?

(1) The cubic has minima at $\sqrt{\frac{\rho}{3}}$ and maxima at $-\sqrt{\frac{\rho}{3}}$

(2) The cubic has minima at $-\sqrt{\frac{\rho}{3}}$ and maxima at $\sqrt{\frac{\rho}{3}}$

(3) The cubic has minima at both $\sqrt{\frac{\rho}{3}}$ and $-\sqrt{\frac{\rho}{3}}$

(4) The cubic has maxima at both $\sqrt{\frac{\rho}{3}}$ and $-\sqrt{\frac{\rho}{3}}$ **[2008]**

18. A value of C for which the conclusion of Mean Value Theorm holds for the function f(x) = $\log_e x$ on the interval [1,3] is

(1) $2\log_3 e$ (2) $\frac{1}{2}\log_e 3$

(3) $\log_3 e$ (4) $\log_e 3$ **[2007]**

19. The function f(x) = $\tan^{-1}(\sin x + \cos x)$ is an increasing function in

(1) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (2) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

(3) $\left(0, \frac{\pi}{2}\right)$ (4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **[2007]**

20. A body falling from rest under gravity passes a certain point P. It was at a distance of 400m from P, 4s prior to passing through P. If g = 10 m/s^2 , then the height above the point P from where the body began to fall is

(1) 720m (2) 900m

(3) 320m (4) 680m **[2006]**

21. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at

(1) x = 2 (2) x = -2

(3) x = 0 (4) x = 1 **[2006]**

22. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2,0) and (3,0) is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$ **[2006]**

23. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(1) 2ab (2) ab

(3) $\sqrt{ab}$ (4) $\frac{a}{b}$ **[2005]**

24. The normal to the curve $x = a(\cos\theta + \theta\sin\theta)$, y = $a(\sin\theta - \theta\cos\theta)$ at any point 'θ' is such that

(1) If passes through the origin

(2) If makes angle $\frac{\pi}{2} + \theta$ with the x – axis

(3) It passes through $\left(a\frac{\pi}{2}, -a\right)$

(4) It is at a constant distance from the origin **[2005]**

25. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched?

	Interval	**Function**
(1)	$(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(2)	$[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(3)	$\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$
(4)	$[-\infty, -4]$	$x^3 + 6x^2 + 6$

[2005]

26. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness, then melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases , is

(1) $\frac{1}{36\pi} cm/\min$ (2) $\frac{1}{18\pi} cm/\min$

(3) $\frac{1}{54\pi} cm/\min$ (4) $\frac{5}{6\pi} cm/\min$ **[2005]**

27. A lizard, at an initial distance of 21 cm behind an insect, moves from rest with an acceleration of 2 cm/s^2 and pursues the insect which is crawling uniformly along a straight line at a speed of 20 cm/s. Then the lizard will catch the insect after

(1) 20 s (2) 1 s

(3) 21 s (4) 24 s **[2005]**

28. Two points A and B move from rest along a straight line with constant acceleration f and f' respectively. If A takes m sec. more than B and describes 'n' units more than B in acquiring the same speed then

(1) $(f-f')m^2 = ff'n$ (2) $(f + f')m^2 = ff'n$

(3) $\frac{1}{2}(f+f')m = ff'n^2$ (4) $(f' - f)n = \frac{1}{2} ff'm^2$ **[2005]**

29. A particle is projected from a point O with velocity u at an angle of 60^o with the horizontal. When it is moving in a direction at right angles to its direction at O, its velocity then is given by

(1) u/3 (2) u/2

(3) 2u/3 (4) $u/\sqrt{3}$ **[2005]**

30. If the equation

$a_nx^n + a_{n-1}x^{n-1} + \ldots\ldots + a_1x = 0$, $a1 \neq 0$, $n \geq 2$, has a positive root $x = \alpha$, then the equation $na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots\ldots + a_1 = 0$ has a positive root, which is

(1) Greater than α (2) smaller than α

(3) greater than or equal to α (4) equal to α **[2005]**

31. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(1) (2,4) (2) (2,-4)

(3) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (4) $\left(\frac{9}{8}, \frac{9}{2}\right)$ **[2004]**

32. A function y = f(x) has a second order derivative f''(x) = 6(x-1). If its graph passes through the point (2,1) and at that point the tangent to the graph is y = 3x – 5, then the function is

(1) $(x-1)^2$ (2) $(x-1)^3$

(3) $(x+1)^3$ (4) $(x+1)^2$ **[2004]**

33. The normal to the curve x = a(1 + cosθ), y = a sinθ at 'θ' always passes through the fixed point

(1) (a,0) (2) (0,a)

(3) (0,0) (4) (a,a) **[2004]**

34. If 2a + 3b + 6c = 0, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval

(1) (0,1) (2) (1,2)

(3) (2,3) (4) (1,3) **[2004]**

35. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where a > 0, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

(1) 3 (2) 1

(3) 2 (4) ½ **[2003]**

36. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{u}$ and the other from rest with uniform acceleration $\vec{f}$. Let α be the

angle between their directions of motion. The relative velocity of the second particle with respect to the first is least after a time

(1) $\frac{u\sin\alpha}{f}$ (2) $\frac{f\cos\alpha}{u}$

(3) u sin α (4) $\frac{u\cos\alpha}{f}$ **[2003]**

37. Two stones are projected from the top of a cliff h meters high, with the same speed u so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected at an angle θ to the horizontal then tan θ equals

(1) $\sqrt{\frac{2u}{gh}}$ (2) $2g\sqrt{\frac{u}{h}}$

(3) $2h\sqrt{\frac{u}{g}}$ (4) $u\sqrt{\frac{2}{gh}}$ **[2003]**

38. A body travels a distance s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. The value of t is given by

(1) $2s\left(\frac{1}{f}+\frac{1}{r}\right)$ (2) $\frac{2s}{\frac{1}{f}+\frac{1}{r}}$

(3) $\sqrt{2s(f+r)}$ (4) $\sqrt{2s\left(\frac{1}{f}+\frac{1}{r}\right)}$ **[2003]**

39. If 2a + 3b + 6c = 0 (a, b, c ε R), then the quadratic equation $ax^2+bx+c = 0$ has

(1) atleast one root in [0,1] (2) atleast one root in [2,3]

(3) atleast one root in [4,5] (4) none of these **[2002]**

40. f(x) and g(x) are two differentiable functions on [0,2] such that f''(x) – g''(x) = 0 , f'(1) = 2g'(1) = 4, f(2) = 3 g(2) = 9 , then f(x) – g(x) at x = 3/2 is

(1) 0 (2) 2

(3) 10 (4) 5 **[2002]**

41. The maximum distance from origin of a point on the curve x = a sin t – b sin $\left(\frac{at}{b}\right)$, y = cos t – b cos $\left(\frac{at}{b}\right)$, both a, b > 0 is

(1) a – b (2) a + b

(3) $\sqrt{a^2+b^2}$ (4) $\sqrt{a^2-b^2}$ **[2002]**

Assertion – Reason Type

1. Consider the function, f(x) = |x-2|+ |x-5|, $x \in R$

   Statement – I : f'(4) = 0

   Statement – **II** : f is continuous in [2,5], differentiable in (2,5) and f(2) = f(5)

2. Let a, b ε R be such that the function f given by f(x) = ln |x| + bx^2+ax, $x \neq 0$ has extreme values at x = -1 and x = 2.

   Statement – I : f has local maximum at x = -1 and x = 2

   Statement – **II** :a = ½ and b = -1/4.

3. Let f be a function defined by f(x) = $\begin{cases} \dfrac{\tan x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

   Statement-I :x = 0 is point of minima of f.

   Statement –II :f'(0) = 0.