PROBABILITY

Some Definitions :

Experiment : A operation which can produce some well defined outcomes is known as an experiment.

Random experiment : If in each trail of an experiment conducted under identical conditions, the outcome is not unique, then such an experiment is called a random experiment.

Sample space : The set of all possible outcomes in an experiment is called a sample space. For example, in a throw of dice, the sample space is {1, 2, 3, 4, 5, 6}. Each element of a sample space is called a sample point.

Event :

An event is a subset of a sample space.

Simple event : An event containing only a single sample point is called an elementary or simple event. Events other than elementary are called composite or compound or mixed events.

For example, in a single toss of coin, the event of getting a head is a simple event.

Here S = {H, T} and E = {H}

In a simultaneous toss of two coins, the event of getting at least one head is a compound event.

Here S = {HH, HT, TH, TT} and E = {HH, HT, TH}

Equally likely events : The given events are said to be equally likely, if none of them is expected to occur in preference to the other.

Mutually exclusive events : If two or more events have no point in common, the events are said to be mutually exclusive. Thus E_1 and E_2 are mutually exclusive in $E_1 \cap E_2 = \phi$.

The events which are not mutually exclusive are known as compatible events.

Exhaustive events : A set of events is said to be totally exhaustive (simply exhaustive), if no event out side this set occurs and at least one of these event must happen as a result of an experiment.

Independent and dependent events : If there are events in which the occurrence of one does not depend upon the occurrence of the other, such events are known as independent events. On the other hand, if occurrence of one depend upon other, such events are known as dependent events.

Probability :

In a random experiment, let S be the sample space and $E \subseteq S$, then E is an event.

The probability of occurrence of event E is defined as

$$P(E) = \frac{\text{number of distinct elements in E}}{\text{number of distinct element in S}} = \frac{n(E)}{n(S)}$$

$$= \frac{\text{number of outocomes favourable to occurrence of E}}{\text{number of all possible outcomes}}$$

Notations :

Let A and B be two events, then

- $A \cup B$ or $A + B$ stands for the occurrence of at least one of A and B.
- $A \cap B$ or AB stands for the simultaneous occurrence of A and B.
- $A' \cap B'$ stands for the non-occurrence of both A and B.
- $A \subseteq B$ stands for "the occurrence of A implies occurrence of B".

Random variable :

A random variable is a real valued function whose domain is the sample space of a random experiment.

Bay's rule :

Let (H_j) be mutually exclusive events such that $P(H_j) > 0$ for j = 1, 2, ... n and $S = \bigcup_{j=1}^{n} H_j$. Let A be an events with $P(A) > 0$, then for j = 1, 2, ... , n

$$P\left(\frac{H_j}{A}\right) = \frac{P(H_j)P(A/H_j)}{\sum_{k=1}^{n} P(H_k)P(A/H_k)}$$

Binomial Distribution :

If the probability of happening of an event in a single trial of an experiment be p, then the probability of happening of that event r times in n trials will be ${}^nC_r\, p^r (1-p)^{n-r}$.

Some important results :

(A) • $P(A) = \frac{\text{Number of cases favourable to event A}}{\text{Total number of cases}}$

$$= \frac{n(A)}{n(S)}$$

- $P(\overline{A}) = \dfrac{\text{Number of cases not favourable to event A}}{\text{Total number of cases}}$

  $= \dfrac{n(\overline{A})}{n(S)}$

(B) Odd in favour and odds against an event : As a result of an experiment if "a" of the outcomes are favourable to an event E and b of the outcomes are against it, then we say that odds are a to b in favour of E or odds are b to a against E.

Thus odds in favour of an event E

$$= \frac{\text{Number of favourable cases}}{\text{Number of unfavourable cases}} = \frac{a}{b}$$

Similarly, odds against an event E

$$= \frac{\text{Number of unfavourable cases}}{\text{Number of favorable cases}} = \frac{b}{a}$$

Note :

- If odds in favour of an event are a : b, then the probability of the occurrence of that event is $\frac{a}{a+b}$ and the probability of non-occurrence of that event is $\frac{b}{a+b}$.
- If odds against an event are a : b, then the probability of the occurrence of that event is $\frac{b}{a+b}$ and the probability of non-occurrence of that event is $\frac{a}{a+b}$.

(C)
- $P(A) + P(\overline{A}) = 1$
- $0 \le P(A) \le 1$
- $P(\phi) = 0$
- $P(S) = 1$
- If $S = \{A_1, A_2, ... A_n\}$, then
  $P(A_1) + P(A_2) + ... + P(A_n) = 1$
- If the probability of happening of an event in one trial be p, then the probability of successive happening of that event in r trials is p^r.

(D)
- If A and B are mutually exclusive events, then
  $P(A \cup B) = P(A) + P(B)$ or
  $P(A + B) = P(A) + P(B)$
- If A and B are any two events, then
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ or
  $P(A + B) = P(A) + P(B) - P(AB)$
- If A and B are two independent events, then
  $P(A \cap B) = P(A) . P(B)$ or
  $P(AB) = P(A) . P(B)$
- If the probabilities of happening of n independent events be $p_1, p_2, ... , p_n$ respectively, then

  (i) Probability of happening none of them
  $= (1 - p_1)(1 - p_2) .... (1 - p_n)$

  (ii) Probability of happening at least one of them
  $= 1 - (1 - p_1)(1 - p_2) .... (1 - p_n)$

  (iii) Probability of happening of first event and not happening of the remaining
  $= p_1(1 - p_2)(1 - p_3) .... (1 - p_n)$
- If A and B are any two events, then

  $P(A \cap B) = P(A) . P\left(\frac{B}{A}\right)$ or

  $P(AB) = P(A) . P\left(\frac{B}{A}\right)$

  Where $P\left(\frac{B}{A}\right)$ is known as conditional probability means probability of B when A has occured.
- **Difference between mutually exclusiveness and independence :** Mutually exclusiveness is used when the events are taken from the same experiment and independence is used when the events are taken from the same experiments.

(E)
- $P(A\overline{A}) = 0$
- $P(AB) + P(\overline{AB}) = 1$
- $P(\overline{A}B) = P(B) - P(AB)$
- $P(A\overline{B}) = P(A) - P(AB)$
- $P(A + B) = P(A\overline{B}) + P(\overline{A}B) + P(AB)$

Some important remark about coins, dice and playing cards :

- **Coins :** A coin has a head side and a tail side. If an experiment consists of more than a coin, then coins are considered to be distinct if not otherwise stated.
- **Dice :** A die (cubical) has six faces marked 1, 2, 3, 4, 5, 6. We may have tetrahedral (having four faces 1, 2, 3, 4,) or pentagonal (having five faces 1, 2, 3, 4, 5) die. As in the case of coins, If we have more than one die, then all dice are considered to be distinct if not otherwise stated.
- **Playing cards :** A pack of playing cards usually has 52 cards. There are 4 suits (Spade, Heart, Diamond and Club) each having 13 cards. There are two colours red (Heart and Diamond) and black (Spade and Club) each having 26 cards.

In thirteen cards of each suit, there are 3 face cards or coart card namely king, queen and jack. So there are in all 12 face cards (4 kings, 4 queens and 4 jacks). Also there are 16 honour cards, 4 of each suit namely ace, king, queen and jack.