INVERSE TRIGNOMETRY

- Meaning of inverse function :
  1. $\sin\theta = x \Leftrightarrow \sin^{-1}x = \theta$
  2. $\cos\theta = x \Leftrightarrow \cos^{-1}x = \theta$
  3. $\tan\theta = x \Leftrightarrow \tan^{-1}x = \theta$
  4. $\cot\theta = x \Leftrightarrow \cot^{-1}x = \theta$
  5. $\sec\theta = x \Leftrightarrow \sec^{-1}x = \theta$
  6. $\mathrm{cosec}\,\theta = x \Leftrightarrow \mathrm{cosec}^{-1}x = \theta$
- Domains and Range of Functions :

Function	Domain	Range
$\sin^{-1}x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\cos^{-1}x$	$-1 \le x \le 1$	$0 \le \theta \le \pi$
$\tan^{-1}x$	$-\infty < x < \infty$, i.e. $x \in R$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$\mathrm{cosec}^{-1}x$	$x \le -1, x \ge 1$	$\theta \ne 0, -\frac{\pi}{2} \le \theta < \frac{\pi}{2}$
$\sec^{-1}x$	$x \le -1, x \ge 1$	$\theta \ne \frac{\pi}{2}, 0 \le \theta \le \pi$
$\cot^{-1}x$	$-\infty < x < \infty$ i.e. $x \in R$	$0 < \theta < \pi$

- **Properties of Inverse Functions :**

(a) 1. $\sin^{-1}(\sin\theta) = \theta$, $\sin(\sin^{-1}x) = x$

2. $\cos^{-1}(\cos\theta) = \theta$, $\cos(\cos^{-1}x) = x$
3. $\tan^{-1}(\tan\theta) = \theta$, $\tan(\tan^{-1}x) = x$
4. $\cot^{-1}(\cot\theta) = \theta$, $\cot(\cot^{-1}x) = x$
5. $\sec^{-1}(\sec\theta) = \theta$, $\sec(\sec^{-1}x) = x$
6. $\mathrm{cosec}^{-1}(\mathrm{cosec}\,\theta) = \theta$, $\mathrm{cosec}(\mathrm{cosec}^{-1}x) = x$

(b) 1. $\sin^{-1}x = \mathrm{cosec}^{-1}(1/x)$

2. $\cos^{-1}x = \sec^{-1}(1/x)$
3. $\tan^{-1}x = \cot^{-1}(1/x)$

(c) 1. $\sin^{-1}(-x) = -\sin^{-1}x$

2. $\cos^{-1}(-x) = \pi - \cos^{-1}x$
3. $\tan^{-1}(-x) = -\tan^{-1}x$
4. $\cot^{-1}(-x) = \pi - \cot^{-1}x$
5. $\sec^{-1}(-x) = \pi - \sec^{-1}x$
6. $\mathrm{cosec}^{-1}(-x) = -\mathrm{cosec}^{-1}x$

(d). 1. $\sin^{-1}x + \cos^{-1}x = \pi/2$

2. $\tan^{-1}x + \cot^{-1}x = \pi/2$
3. $\sec^{-1}x + \mathrm{cosec}^{-1}x = \pi/2$

- **Formulae for Sum and Difference of Inverse Function –**

1. $\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\dfrac{x+y}{1-xy} & \text{where } xy < 1 \\ \pi + \tan^{-1}\dfrac{x+y}{1-xy} & \text{when } xy > 1 \end{cases}$
2. $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\dfrac{x-y}{1+xy}$
3. $\sin^{-1}x \pm \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right\}$
4. $\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\left\{xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right\}$
5. $\cot^{-1}x \pm \cot^{-1}y = \cot^{-1}\left[\dfrac{xy \mp 1}{y \pm x}\right]$
6. $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\dfrac{x+y+z-xyz}{1-xy-yz-zx}\right]$

- **Some Important Results :**

1. $2\sin^{-1}x = \sin^{-1}2x\sqrt{1-x^2}$
2. $2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$
3. $2\tan^{-1}x = \tan^{-1}\dfrac{2x}{1-x^2} = \sin^{-1}\dfrac{2x}{1+x^2} = \cos^{-1}\dfrac{1-x^2}{1+x^2}$
4. $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$
5. $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$
6. $3\tan^{-1}x = \tan^{-1}\dfrac{3x - x^3}{1-3x^2}$
7. $\tan^{-1}\left[\dfrac{x}{\sqrt{a^2 - x^2}}\right] = \sin^{-1}\left(\dfrac{x}{a}\right)$
8. $\tan^{-1}\left[\dfrac{3a^2x - x^3}{a(a^2 - 3x^2)}\right] = 3\tan^{-1}\left(\dfrac{x}{a}\right)$
9. $\tan^{-1}\left[\dfrac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right] = \dfrac{\pi}{4} + \dfrac{1}{2}\cos^{-1}x^2$